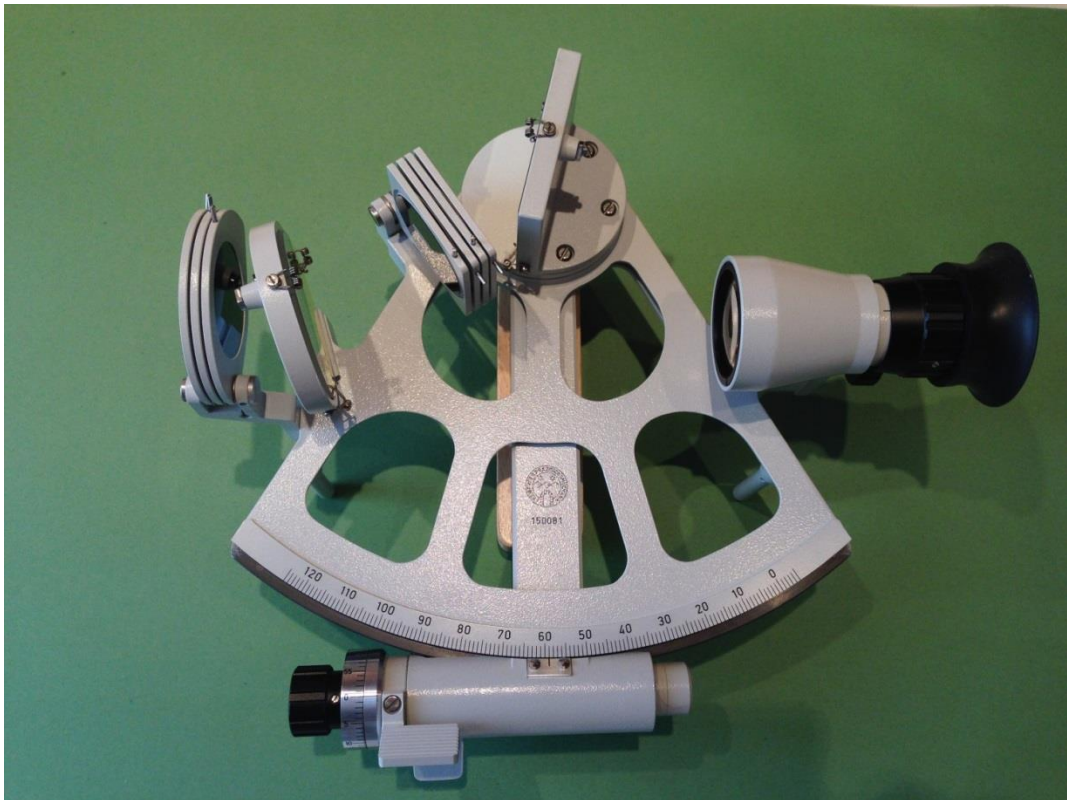


# Astronavigation

with

## Sextant and Calculator HP-42S



## Abbreviations and symbols

$LOP1$  = Line of position 1

$LOP2$  = Line of position 2

$COP1$  = Circle of position 1

$COP2$  = Circle of position 2

$H1$  = Observed altitude ( $H_o$ ) in  $COP1$

$H2$  = Observed altitude ( $H_o$ ) in  $COP2$

$Dec$  = Declination

$GHA$  = Greenwich Hour Angle

$D1$  =  $Dec$  of the sun in  $COP1$

$G1$  =  $GHA$  of the sun in  $COP1$

$D2$  =  $Dec$  of the sun in  $COP2$

$G2$  =  $GHA$  of the sun in  $COP2$

$\varphi$  = Latitude of the  $FIX$  in the calculations of the COP

$L$  = Longitude of the  $FIX$  in the calculations of the COP

$\Delta\varphi$  = Displacement in latitude of the observer

$\Delta L$  = Displacement in longitude of the observer

$lat_M$  = Average latitude

$latx$  = Dead-reckoning or assumed observer's latitude

$lonx$  = Dead-reckoning or assumed observer's longitude

$FIX$  = Calculated position

$laf$  =  $FIX$  latitude

$lof$  =  $FIX$  longitude

$IN1, IN2$  = Intercept of  $LOP1, LOP2$

$AZ1, AZ2$  = Azimuth of  $LOP1, LOP2$

$Ha$  = Apparent altitude

$Dog$  = Distance over Ground of the observer

$Cog$  = Course over Ground of the observer

$Ie$  = Index error of the sextant

*HoE* = Height of eye of the observer

*Da. Mo* = Day and Month of measurement

*Year* = Year of measurement

*U. T* = Universal Time of measurement

*ALTs* = Measured sextant height

*Limb* = Top edge, bottom edge or center line of the sun

*Npos* = Northern intersection of the circles of equal altitude

*Spos* = Southern intersection of the circles of equal altitude

*Tran* = Noon latitude of the observer (Transit)

## **Foreword**

The astronomical positioning by the simultaneous measurement of a number of stars leads to a fairly accurate position, but the exercise necessary to identify those stars and to determine their height above the horizon at twilight will often be absent. That is why we limit ourselves to determine our position with the help of the sun.

Using nautical almanacs such as the Macmillan and the HO249 tables, we can traditionally determine our position on the basis of sun's altitudes.

With the advent of the programmable calculators, the entire search, calculation and plot work that follows could be taken over from us. *GHA* and *Dec* of the sun can be calculated with a built-in algorithm, as well as the azimuth and intercept of the lines of position. The somewhat difficult chart work is also digitally reproduced with the final result our position in latitude and longitude (*FIX*).

## **HP-42S**

The HP-42S RPN Scientific is a programmable RPN calculator launched by HP in 1988. Its production was discontinued in 1995 but it is still regarded today as one of the best ever made in terms of quality and ease of programming. His popularity was so great that he is offered on the site of [Thomas Okken](#) under the name "**Free42**: An HP-42S Calculator Simulator" as freeware for many operating systems of *computers and smartphones*. The Free42 computer application is delivered as a zip file in decimal and binary version but it is recommended to install Free42Decimal.exe. Android and iOS applications are in decimal version.

The Free42 app has the advantage that the programs can be saved and loaded as files, backups can be made and loaded into other computers. Instructions can be found [here](#). Moreover, the calculation speed is many times higher than with the hardware version of the HP-42S.

Free42 programs are saved with the file extension ".raw".

In the astronavigation programs described below, only a limited number of keys is used, so that no thorough knowledge of the calculator is required. Those who want to deepen further can go [here](#).

## Programs

This text describes two programs for Free42 and HP-42S to navigate by the sun by means of a sextant or theodolite:

*ASTRO\_TSO.raw*: calculates the position (*FIX*) using the azimuth-intercept method of Marcq St. Hilaire. This method, developed in 1875, gradually became the standard procedure for astronomical positioning and is based on the choice of an assumed position (A.P), as close as possible to the true position or the estimated position, and the calculation of the intersection of two so-called *lines of position* (LOP further in the text). This program makes the use of maps and plotting sheets unnecessary.

*ASTRA\_TSO.raw*: calculates the position (*FIX*) using an alternative method that directly calculates the intersections of two *circles of position* (COP further in the text). For a *stationary* observer, contrary to the previous method, no assumed position (A.P) is required so that one can determine his position without any idea of where one is. In the case of a *moving observer*, as in the previous program, the latitude of the observer, *latx*, must be known for the calculation of the displacement in longitude  $\Delta L$ .

Both programs have a built-in almanac of the sun with a maximum deviation of  $\pm 0.5'$  and an average deviation of  $\pm 0.3'$  for *GHA* and *Dec*. For verification, the almanac data on the sites [backbearing.com](http://backbearing.com) and [siranah.de](http://siranah.de) can be consulted.

## Corrections to the measured sextant height *ALTs*

A number of corrections are applied to the measured sextant height in both programs;

- *Instrument correction*: is assumed to be = 0 in both programs. If the sextant is provided with a certificate, this correction can be taken into account for the input of *ALTs*.
- *Index correction*: is a correction for the index error *Ie* of the sextant which in principle has to be measured before each new observation. *Ie* is negative if "off the arc" and positive "on the arc".
- *Dip*: this is a correction that is function of the height of eye *HoE*. When using an artificial horizon *HoE* = 0.

$$Dip['] \approx 1.76 \cdot \sqrt{HoE[m]}$$

- *Refraction*: this is a correction for the refraction of light according to Bennett's formula.

$$Refractie['] = \frac{1}{\tan\left(Ha[^\circ] + \frac{7.31}{Ha[^\circ] + 4.4}\right)}$$

- *Parallax*: as the horizontal parallax of the sun is very small (maximum  $\pm 0.15'$ ), no correction for parallax is applied.
- *Semi diameter (S.D)*: this correction is made if the bottom or top edge (*Limb*) of the sun is measured. Between summer and winter the *S.D* varies between  $\pm 15.75'$  and  $16.29'$ , and in the programs the average  $16' = 0.2667^\circ$  is used. For greater accuracy, the value in the program line #326 in ASTRO and #282 in ASTRA must be replaced by the actual value (in degrees!) of the [sun-almanac](http://sun-almanac.com).

## Sextant heights

$Ha = ALTs - Ie - Dip$  Apparent altitude

$Ho = Ha - Refractie - S.D$  Observed altitude

$HoE = 0$  implies the use of an artificial horizon

When using an artificial horizon  $Dip = 0$  and  $Ha = (ALTs - Ie)/2$

## Used formulas

- If the  $Cog$  and  $Dog$  are known then the observer's displacement in latitude and longitude are respectively:

$$\Delta\varphi[^\circ] = Dog[nm] \cdot \cos Cog / 60$$

$$\Delta L[^\circ] = \frac{Dog[nm] \cdot \sin Cog}{\cos lat_M \cdot 60}$$

in which the average latitude

$$lat_M = latx + \Delta\varphi/2$$

and  $latx$  is the latitude before displacement.

- In *ASTRO\_TSO.raw* (azimuth-intercept method) the *FIX*-coordinates are

$$laf = \frac{IN2 \cdot \sin AZ1 - IN1 \cdot \sin AZ2}{\sin(AZ1 - AZ2)} + latx$$

$$lof = \frac{IN1 \cdot \cos AZ2 - IN2 \cdot \cos AZ1}{\sin(AZ1 - AZ2) \cdot \cos latx} + lonx$$

where  $latx$  and  $lonx$  are the coordinates of the A.P after displacement. These values are calculated from the A.P before displacement,  $Cog$  and  $Dog$ .

## Format and units of the input data

$latx, lonx$		+ (N and E) – (S and W)
$Ie$	x.xx [minutes]	– OFF the arc + ON the arc
$HoE$	xx.xx [meter]	Artificial horizon $HoE=0$
$Da.Mo$	xx.xx [day.month]	Ex: 5.07 23.12
$U.T$	xx.xx xx [h.min sec]	Ex: 17.0533 6.0806
$latx, lonx, ALTs$	xxx.xx x [ddd°mm.m']	Ex: –15.416 156.093
$Limb$	Lower=-1 Center=0 Upper=1	
$Dog$	xxx.x [nautical miles]	
$Cog$	xxx.x [degrees]	

## Program components

**ASTRO\_TSO.raw**: consists of 3 subroutines; *LOP1*, *LOP2*, and *TRAN*

- *LOP1 (Line of position1)*: here the assumed position (A.P), which is usually the dead-reckoning position, and the data of the first observation are entered. The subroutine calculates the intercept *IN1* and azimuth *AZ1* of the first line of position *LOP1* and stores them in their variables with the same name.
- *LOP2 (Line of position2)*: on the basis of the *Dog* and *Cog* a new A.P is defined. Together with the data of the second observation, the *IN2* and *AZ2* of *LOP2* are calculated relative to this new A.P. The coordinates of the intersection of the "offset" *LOP1* and *LOP2*, being the *FIX*, are obtained with the above formulas. From a geometric point of view this actually amounts to the translation of *LOP1* in the direction (= *Cog*) of the new A.P over a distance = *Dog*. The coordinates of the *FIX* are stored as "*laf*" and "*lof*".  
Since the intercept method ignores the curvatures of the actual position lines, the obtained *FIX* is not our exact position but rather an improved position (compared with AP). The residual error remains tolerable as long as the radii of the circles of equal altitude are not too small and AP is not too far from our actual position. The geometric error inherent to the intercept method can be decreased by iteration, i.e., substituting the obtained *FIX* for AP and repeating the calculations with the same altitudes and GP's (see appendix). This will result in a more accurate position. If necessary, we can reiterate the procedure until the obtained position remains virtually constant (rarely needed).
- *TRAN (Transit)*: this part calculates the noon latitude of our position. The input of the estimated longitude *lonx* only results in a rough estimate of the time of transit because it is calculated on the basis of a fixed transit time of the Greenwich meridian (12h U.T). It is more accurate to start from the actual "[Greenwich meridian transit time](#)" and to add 4 minutes per degree of western length or subtract 4 minutes per degree of eastern length.  
"South" and "North" indicate the position of the sun relative to the observer.  
The noon latitude is stored as "*laf*".

### Notes

- *LOP1* and *LOP2* must be entered chronologically

- when using a theodolite,  $HoE = 0$  should be set, in analogy with an artificial horizon. To prevent *Ha* from being divided by 2, set  $HoE = 0.0001$  and the *Dip* correction applied by this is negligible.

**ASTRA\_TSO.raw** : consists of 3 subroutines; *COP1*, *COP2*, and *TRAN*

- *COP1 (Circle of position1)*: *latx* is the estimated or known latitude of the first observation and will be used in *COP2* to calculate the displacement in longitude  $\Delta L$ . For a stationary observation ( $Dog = 0$ ) this value is irrelevant.  
After inputting the other measured values, this routine calculates the parameters that define the *Circle of position1*, i.e. *GHA* (= *G1*), *Dec* (= *D1*) and *Ho* (= *H1*).
- *COP2 (Circle of position2)*: with *Dog* and *Cog* we calculate the displacement of the first *circle of position COP1*, being  $\Delta\phi$  in latitude and  $\Delta L$  in longitude. With the other measured values, this routine calculates the parameters that define the *Circle of position2*, i.e. *GHA* (= *G2*), *Dec* (= *D2*), *Ho* (= *H2*), and ultimately our position (*FIX*).

These 2 circles normally have 2 intersections on the globe. These are our 2 possible positions. When calculating the coordinates of these points of intersection, the choice is made between the northern intersection point (*Npos*) and the southern intersection point (*Spos*). The coordinates of the *FIX* are stored as "*laf*" and "*lof*".

The algorithm for calculating the intersections is discussed in detail in the appendix.

- *TRAN (Transit)*: see *ASTRO\_TSO.raw*

Notes: see *ASTRO\_TSO.raw*

## ASTRO versus ASTRA

- If one is completely in the dark about his position one can not directly enter a suitable A.P into ASTRO. In case of a *stationary measurement* ( $Dog=0$ ), ASTRA does not need this data. With 2 observations, preferably with a difference in azimuth between  $30^\circ$  and  $150^\circ$ , an accurate position can be determined directly and apart from the normal measurement errors



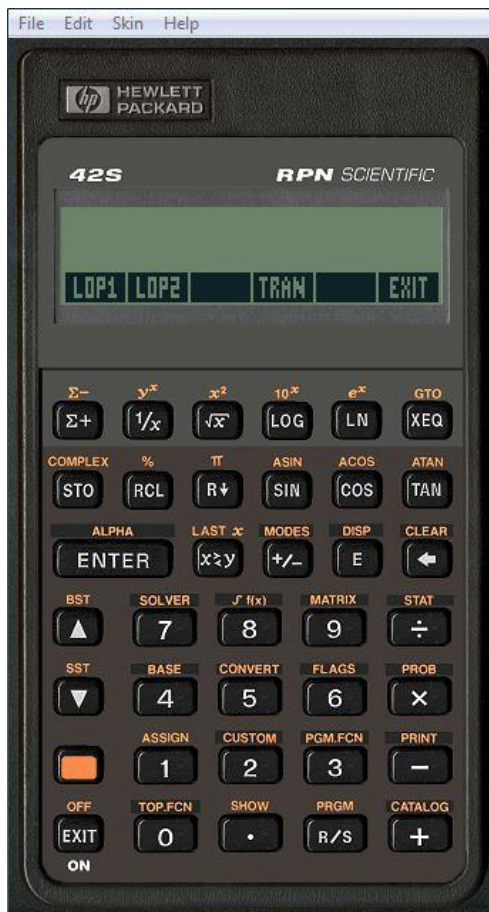
- The iteration process of the ASTRA-program runs very fast in Free42 but can take a lot of time with the HP-42S under certain circumstances ( $H1 \approx H2$ ). due to its low clock speed.

- ASTRA always iterates to the most accurate position. With ASTRO, at a great distance between the A.P and the true position and / or at large sextant heights, at least 1 iteration may be required with the calculated *FIX* as the new A.P.

## Free42 and HP-42S commands

- Press the **XEQ** button. The list of programs that have been loaded appears in the display.

- Press the key below the name of the selected program and after viewing the label, press **R/S**. The main menu appears in the display, consisting of the 3 subroutines and the **EXIT** routine to close the program.



- Each routine is started with the key under its name. The requested data is entered via the number pad. Pay particular attention to the units and the format. For negative numbers, first enter the number followed by  $\boxed{+/-}$ . An incorrect input can be deleted with the  $\boxed{\leftarrow}$  key and with  $\boxed{\text{RTN}}$  you return to the main menu. Each input is followed by  $\boxed{\text{R/S}}$ . In a selection menu, press the key below the desired choice. If the program stops with relevant information in the display, press  $\boxed{\text{R/S}}$  to continue.

- Entered data and calculated values are stored as variables and can be called up with  $\boxed{\text{RCL}}$  and scroll keys  $\boxed{\uparrow}$  and  $\boxed{\downarrow}$

- Calculated values *GHA*, *Dec*, *lat* and *lon* are stored in the format [ddd°mm.m']. *AZ1* and *IN1* in [°].

- The results of the observations can be printed out. The print function is activated by  $\boxed{\text{PRINT}} \boxed{\uparrow} \boxed{\text{PON}}$  and disabled by  $\boxed{\text{PRINT}} \boxed{\uparrow} \boxed{\text{POFF}}$ . The print is available for almost all operating systems via the File menu. With the iPhone the screen must be touched just below the status bar.

## Appendix: calculation of the intersections of 2 circles of position

We apply the formulas of the nautical triangle to 2 observations;

$$\sin H1 = \sin D1 \cdot \sin(\varphi - \Delta\varphi) + \cos D1 \cdot \cos(\varphi - \Delta\varphi) \cdot \cos(G1 + L - \Delta L) \quad (1)$$

$$\sin H2 = \sin D2 \cdot \sin \varphi + \cos D2 \cdot \cos \varphi \cdot \cos(G2 + L) \quad (2)$$

(1) and (2) are the equations of 2 circles of position in which  $\varphi$  and  $L$  are the geographical coordinates of the position of the observer at the time of the second measurement, i.e. the *FIX*.  $H1$  and  $H2$  are the true sextant heights (after corrections) and  $\Delta\varphi$  and  $\Delta L$  are respectively the displacement in latitude and longitude after the first measurement.

The system of 2 equations with 2 unknowns ( $\varphi$  and  $L$ ) can not be solved analytically but by iteration. There are 2 conditions attached to this process:

- the iteration must be started in the equation of the circle with the smallest sextant height  $H$ , i.e. the circle with the largest radius.

- since for 1 value of  $\varphi$ , 2 values of  $L$  are possible and vice versa the appropriate choice must be made here.

If  $\boxed{H2 < H1}$

then the starting value of  $\varphi$  (=  $D2$ ) is placed in equation (2). This gives 2 possible  $L$ -values:

$L = \text{Acos}(\dots) - G2$  and  $L = -\text{Acos}(\dots) - G2$ . One chooses the most eastern (= positive) value and since  $\text{Acos}$  is always positive

$$L = \text{Acos}\left(\frac{\sin H2 - \sin D2 \cdot \sin \varphi}{\cos D2 \cdot \cos \varphi}\right) - G2$$



This L-value is placed in (1) and the new  $\varphi$ -value for (2) is calculated

$$\varphi = 2 \cdot \text{Atan} \left( \frac{\sin D1 \pm \sqrt{\sin^2 D1 - \sin^2 H1 + \cos^2 D1 \cdot \cos^2 (G1 + L - \Delta L)}}{\sin H1 + \cos D1 \cdot \cos (G1 + L - \Delta L)} \right) + \Delta\varphi$$

with  $+\sqrt{\dots}$  for the northern intersection and  $-\sqrt{\dots}$  for the southern intersection.

If **H2 > H1**

then the starting value of  $\varphi$  (= D2) is placed in equation (1). This gives 2 possible L -values:  
 $L = \text{Acos}(\dots) - G1 + \Delta L$  and  $L = -\text{Acos}(\dots) - G1 + \Delta L$ . One chooses the most western (= negative) value and since Acos is always positive

$$L = -\text{Acos} \left( \frac{\sin H1 - \sin D1 \cdot \sin(\varphi - \Delta\varphi)}{\cos D1 \cdot \cos(\varphi - \Delta\varphi)} \right) - (G1 - \Delta L)$$

This L-value is placed in (2) and the new  $\varphi$ -value for (1) is calculated

$$\varphi = 2 \cdot \text{Atan} \left( \frac{\sin D2 \pm \sqrt{\sin^2 D2 - \sin^2 H2 + \cos^2 D2 \cdot \cos^2 (G2 + L)}}{\sin H2 + \cos D2 \cdot \cos (G2 + L)} \right)$$

with  $+\sqrt{\dots}$  for the northern intersection and  $-\sqrt{\dots}$  for the southern intersection.

In both cases the value of  $\varphi$  is calculated by substitution, by means of the so-called T-formulas, with variables  $\tan((\varphi - \Delta\varphi)/2)$  and  $\tan(\varphi/2)$  respectively.

The iterations are stopped when 2 consecutive values of  $\varphi$  differ by less than  $0.001^\circ$

In the choices made in the iterative process described above, it is implicitly assumed that the geographic position (G.P) of the sun at the second observation lies to the west of the G.P of the first observation.

## Appendix: iterations in ASTRO

A "running" fix is less accurate than a stationary fix. After all, the *Cog* and *Dog* can only be estimated because the effects of current and wind (drift) are not exactly known in most cases. For a stationary measurement (*Dog* = 0), the found position can be refined by inputting the *FIX* as the new A.P with the following key sequence in the main menu:

**LOP1** **RCL** **LAF** **R/S** **RCL** **LOF** **R/S** and repeating the calculations with the initial input parameters.

## Interesting websites

<http://thomasokken.com/free42/>  
<https://www.celnav.de/page2.htm>  
<http://www.teacupnavigation.net/CN.html>  
<http://www.backbearing.com/almanac.html>  
[http://www.siranah.de/html/fr\\_sail.htm](http://www.siranah.de/html/fr_sail.htm)

## List of used equations in the text

$$\sin H1 = \sin D1 \cdot \sin(\varphi - \Delta\varphi) + \cos D1 \cdot \cos(\varphi - \Delta\varphi) \cdot \cos(G1 + L - \Delta L) \quad (1)$$

$$\sin H2 = \sin D2 \cdot \sin \varphi + \cos D2 \cdot \cos \varphi \cdot \cos(G2 + L) \quad (2)$$

start  $\varphi=D2$  in (2)

start  $\varphi=D2$  in (1)

$$L = A\cos\left(\frac{\sin H2 - \sin D2 \cdot \sin \varphi}{\cos D2 \cdot \cos \varphi}\right) - G2$$

$$\varphi = 2 \cdot \text{Atan}\left(\frac{\sin D1 \pm \sqrt{\sin^2 D1 - \sin^2 H1 + \cos^2 D1 \cdot \cos^2(G1 + L - \Delta L)}}{\sin H1 + \cos D1 \cdot \cos(G1 + L - \Delta L)}\right) + \Delta\varphi$$

$$L = -A\cos\left(\frac{\sin H1 - \sin D1 \cdot \sin(\varphi - \Delta\varphi)}{\cos D1 \cdot \cos(\varphi - \Delta\varphi)}\right) - (G1 - \Delta L)$$

$$\varphi = 2 \cdot \text{Atan}\left(\frac{\sin D2 \pm \sqrt{\sin^2 D2 - \sin^2 H2 + \cos^2 D2 \cdot \cos^2(G2 + L)}}{\sin H2 + \cos D2 \cdot \cos(G2 + L)}\right)$$

$$\Delta\varphi = \text{Dog} \cdot \cos \text{Cog} / 60 \quad (\text{decimal } ^\circ)$$

$$\Delta L = \frac{\text{Dog} \cdot \sin \text{Cog}}{\cos \text{lat}_M \cdot 60} \quad (\text{decimal } ^\circ)$$

$$\text{lat}_M = \text{lat}_x + \Delta\varphi/2$$

$$\text{laf} = \frac{IN2 \cdot \sin AZ1 - IN1 \cdot \sin AZ2}{\sin(AZ1 - AZ2)} + \text{lat}_x$$

$$\text{lof} = \frac{IN1 \cdot \cos AZ2 - IN2 \cdot \cos AZ1}{\sin(AZ1 - AZ2) \cdot \cos \text{lat}_x} + \text{lon}_x$$

$$\text{Lon} = \pm A\cos\left(\frac{\sin Ho - \sin Dec \cdot \sin Lat}{\cos Dec \cdot \cos Lat}\right) - GHA$$

$$\text{Dip}['] \approx 1.76 \cdot \sqrt{HoE[m]}$$

$$\text{Refractie}['] = \frac{1}{\tan\left(Ha[^\circ] + \frac{7.31}{Ha[^\circ] + 4.4}\right)}$$